## DAY THIRTEEN

# Behaviour of Perfect Gas and Kinetic Theory

### Learning & Revision for the Day

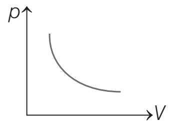
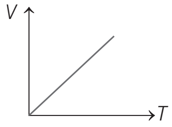
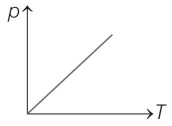
- Ideal Gas or Perfect Gas
- Gas Law
- Equation of State of a Perfect Gas
- Kinetic Theory of Gases
- Various Speeds of Gas Molecule
- Degree of Freedom (f)
- Law of Equipartition of Energy
- Specific Heat Capacities of Gases
- Mean Free Path

## Ideal Gas or Perfect Gas

An ideal gas or perfect gas is that gas which strictly obeys the gas laws such as Boyle's law, Charles' law, Gay Lussac's law, etc. at all value of temperature and pressure.

## Gas Law

Some important gas laws are given below in table form.

| | | |
|---|---|---|
| Boyle's law | $V \propto \frac{1}{p}$ or $pV = \text{constant}$ |  |
| Charles' law | $V \propto T$ or $\frac{V}{T} = \text{constant}$ |  |
| Gay-Lussac's law | $p \propto T$ or $\frac{p}{T} = \text{constant}$ |  |
| Grahm' law of diffusion | $r \propto \frac{1}{\sqrt{\rho}} \propto \frac{1}{\sqrt{M}}$ or $\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{\rho_2}{\rho_1}}$ | |
| Daltan's law of partial pressure | $P = P_1 + P_2 + P_3 + P_4 + \ldots\ldots + P_n$ | |

## Equation of State of a Perfect Gas

The equation which relates the pressure ($P$), volume ($V$) and temperature ($T$) of the given state of an ideal gas is known as ideal or perfect gas equation.

For 1 mole of gas, $\frac{pV}{T} = R$ (constant)

$$pV = RT$$

where, $R$ is **universal gas constant**. The SI unit of gas constant is J/mol-K. Its value is 8.314 J/mol-K or $8.314 \times 10^7$ erg/mol-K or 2 cal/mol-K. The dimensions of $R$ are $[\mathrm{ML^2T^{-1}\theta^{-1}}]$.

Moreover, gas constant $R = \frac{\text{Work}}{\text{Moles} \times \text{Temperature}}$

- The perfect gas equation for 1 molecule of gas is
$$pV = kT$$
- Boltzmann's constant is represented by per mole gas constant
i.e. $k = \frac{R}{N} = \frac{8.31}{6.023 \times 10^{23}} = 1.38 \times 10^{-23}\ \mathrm{JK^{-1}}$
Its dimensions are $[\mathrm{ML^2T^{-2}\theta^{-1}}]$.

## Kinetic Theory of Gases

Kinetic theory of gases relates the macroscopic properties of gases (such as pressure, temperature etc.) to the microscopic properties of gas molecules (such as speed, momentum, kinetic energy of molecules etc).

### Assumptions of Kinetic Theory of Gases

- Every gas is composed of tiny particles known as molecules. The size of molecules is much smaller than the intermolecular spacing.
- The molecules of a gas are identical, spherical, rigid and perfectly elastic point masses.
- Molecules are in a state of random rapid motion. They collide with each other. There is no loss of energy during collision. Only the direction of motion is changed.
- The time spent in collision between two molecules is negligible in comparison to time between two successive collisions.
- The number of collisions per unit volume in a gas remains constant. No attractive or repulsive force acts between gas molecules.
- Gravitational attraction among the molecules is ineffective due to extremely small masses and very high speed of molecules.
- Molecules constantly collide with the walls of container due to which their momentum changes. The change in momentum is transferred to the walls of the container. Consequently, on the walls of container pressure is exerted by gas molecules. The density of gas is constant at all points of the container.

### Kinetic Energy and Temperature

In ideal gases, the point particles can have only translational motion and thus only translational energy.

- Translational KE of a molecule $= \frac{1}{2} m\bar{c}^2 = \frac{3}{2} kT$
- Mean KE per molecule $= \frac{3}{2} kT$
- Mean kinetic energy per gram mole is given by
$$\mathrm{KE_{mole}} = \left(\frac{1}{2} m\bar{c}^2\right) N = \frac{3}{2} kTN = \frac{3}{2} RT$$
- Average kinetic energy of gas $= \frac{3}{2} pV$
- KE per molecule $= \frac{3pV}{2N} = \frac{3RT}{2N}$
- KE per mole $= \frac{3}{2} kT$
- KE per volume $= \frac{3}{2} p$

### Concept of Pressure

Pressure $p$ exerted by a perfect gas on the walls of container is given by

$$p = \frac{1}{3} \frac{mN\bar{c}^2}{V} = \frac{1}{3} \frac{M}{V} \bar{c}^2$$

$$pV = \frac{1}{3}(\rho\bar{c}^2) = \frac{1}{3}\left(\frac{2}{2}\rho\bar{c}^2\right) = \frac{2}{3}\left(\frac{1}{2}\rho\bar{c}^2\right) = \frac{2}{3}E$$

Here, $m =$ mass of each molecule, $\bar{c} =$ root mean square velocity of molecules, $\rho =$ density of gas, $M =$ mass of gas enclosed in volume $V$ of container, and $E =$ Total KE of the ideal gas.

## Various Speeds of Gas Molecules

- **Root mean square speed** It is defined as the square root of mean of squares of the speed of different molecules
i.e. $v_{\mathrm{rms}} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \dots}{N}} = \sqrt{v^2}$
From the expression of pressure, $p = \frac{1}{3} \rho v_{\mathrm{rms}}^2$
$$v_{\mathrm{rms}} = \sqrt{\frac{3p}{\rho}} = \sqrt{\frac{3pV}{\text{Mass of gas}}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}}$$
- **Most probable speed** It is defined as the speed which is possessed by maximum fraction of total number of molecules of the gas.
$$v_{\mathrm{mp}} = \sqrt{\frac{2p}{\rho}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}}$$
- **Average speed** It is the arithmetic mean of the speeds of molecules in a gas at given temperature.
$$v_{\mathrm{av}} = \frac{v_1 + v_2 + v_3 + v_4 + \dots}{N}$$
Average speed, $v_{\mathrm{av}} = \sqrt{\frac{8p}{\pi\rho}} = \sqrt{\frac{8}{\pi} \frac{RT}{M}} = \sqrt{\frac{8kT}{\pi m}}$

## Degree of Freedom ($f$)

The term degree of freedom of a system refers to the possible independent motions a system can have

- for monoatomic gas, $(f) = 3$
- for diatomic gas, $(f) = 5$
- for triatomic gas, $(f) = 6$(non-linear)
- for triatomic (linear) gas, $(f) = 7$
- for $N$-atomic molecule $(f) = 6N - 3$
- for $N$-atomic linear molecule $(f) = 6N - 5$

## Law of Equipartition of Energy

According to **law of equipartition of energy** for any system in thermal equilibrium, the total energy is equally distributed among its various degree of freedom and each degree of freedom is associated with energy $\frac{1}{2}kT$ (where, $k = 1.38 \times 10^{-23}$ J/K and $T$ = absolute temperature of the system).

## Specific Heat Capacities of Gases

The specific heat of gas can have many values, but out of them following two values are important

### Specific Heat at Constant Volume

The specific heat of a gas at constant volume is defined as the quantity of heat required to raise the temperature of unit mass of gas through 1°C or 1 K when its volume is kept constant i.e.

$$c_V = \frac{(\Delta Q)_V}{m\Delta T}$$

For one mole of gas,

$$C_V = Mc_V = \frac{M\,(\Delta Q)_V}{m\Delta T} = \frac{1}{n}\frac{(\Delta Q)_V}{\Delta T} \qquad \left[\because n = \frac{m}{M}\right]$$

### Specific Heat at Constant Pressure

The specific heat of a gas at constant pressure is defined as the quantity of heat required to raise the temperature of unit mass of gas through 1K, when its pressure is kept constant i.e.

$$c_p = \frac{(\Delta Q)_p}{m\Delta T}.$$

For one mole of gas,

$$\therefore \quad C_p = Mc_p = \frac{M\,(\Delta Q)_p}{m\Delta T} \qquad \ldots\text{(i)}$$

$$= \frac{1}{n}\frac{(\Delta Q)_p}{\Delta T}$$

Specific heat of a gas at constant pressure is greater than the specific heat at constant volume i.e. $C_p > C_V$.

$C_p$ and $C_V$ are related to each other according to relation,

$$C_p - C_V = \frac{R}{J} \qquad \ldots\text{(ii)}$$

Eq. (ii) is called **Mayer's relation**.

If $C_p$ and $C_V$ are measured in the units of work and $R$ is also in the units of work (or energy), then Eq. (ii) becomes $C_p - C_V = R$.

### Specific Heat in Terms of Degree of Freedom

For a gas at temperature $T$, the internal energy $U = \frac{f}{2}nRT$.

Change in energy, $\Delta U = \frac{f}{2}nR\Delta T \Rightarrow C_V = \frac{1}{2}fR$

Specific heat at constant pressure, $C_p = \left(\frac{f}{2} + 1\right)R$.

Ratio of $C_p$ and $C_V$, $\gamma = \frac{C_p}{C_V} = \frac{\left(\frac{f}{2} + 1\right)R}{\frac{f}{2}R} = 1 + \frac{2}{f}$.

## Mean Free Path

The distance travelled by a gas molecule between two successive collision is known as free path.

$$\text{Mean free path} = \frac{\text{Total distance covered}}{\text{Number of collisions}}$$

The mean free path of a gas molecule is the average distance between two successive collisions.

It is represented by $\lambda$.

$$\lambda = \frac{1}{\sqrt{2}\,\pi\,\sigma^2 n}$$

Here, $\sigma$ = diameter of the molecule and $n$ = number of molecules per unit volume.

## DAY PRACTICE SESSION 1

# FOUNDATION QUESTIONS EXERCISE

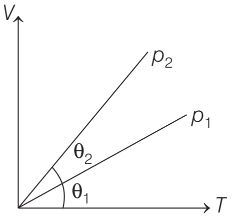
**1** The volume of a given mass of a gas at 27°C, 1 atm is 100 cc. What will be its volume at 327°C?

(a) 50 cc (b) 100 cc (c) 150 cc (d) 200 cc

**2** A vessel contains 1 mole of $O_2$ gas (molar mass 32) at temperature $T$. The pressure of the gas is $p$. An identical vessel containing 1 mole of He gas (molar mass 4) at temperature $2T$ has a pressure of

(a) $\frac{p}{8}$ (b) $p$ (c) $2p$ (d) $8p$

**3** In the given ($V$-$T$) diagram, what is the relation between pressures $p_1$ and $p_2$?



➜ NEET 2013

(a) $p_2 = p_1$ (b) $p_2 > p_1$
(c) $p_2 < p_1$ (d) Cannot be predicted

**4** A balloon is filled at 27°C and 1 atm pressure by 500 $m^3$ He. At – 3°C and 0.5 atm pressure, the volume of He gas will be ➜ AIIMS 2012

(a) 700 $m^3$ (b) 900 $m^3$ (c) 1000 $m^3$ (d) 500 $m^3$

**5** At what temperature, the mean kinetic energy of $O_2$ will be the same as that of $H_2$ molecules at – 73°C?

(a) 127°C (b) 527°C (c) – 73° C (d) –173°C

**6** Relationship between $p, V$ and $E$ for a gas is ($E$ = total translational kinetic energy)

(a) $p = \frac{3}{2}EV$ (b) $V = \frac{2}{3}Ep$ (c) $pV = \frac{3}{2}E$ (d) $pV = \frac{2}{3}E$

**7** When temperature of an ideal gas is increased from 27°C to 227°C. Its rms speed changed from 400 m/s to $v_s$. The $v_s$ is

(a) 516 m/s (b) 450 m/s (c) 310 m/s (d) 746 m/s

**8** The ratio of the speed of sound in nitrogen gas to that in helium gas, at 300 K is

(a) $\sqrt{\left(\frac{2}{7}\right)}$ (b) $\sqrt{\left(\frac{1}{7}\right)}$ (c) $\frac{\sqrt{3}}{5}$ (d) $\frac{\sqrt{6}}{5}$

**9** The molecules of a given mass of gas have rms speed 200 $ms^{-1}$ at 27°C and $10^5$ $Nm^{-2}$ pressure. When the absolute temperature is doubled and the pressure is halved, the rms speed of the molecules of the same gas is

(a) 200 $ms^{-1}$ (b) 400 $ms^{-1}$
(c) 200 $\sqrt{2}$ $ms^{-1}$ (d) 400 $\sqrt{2}$ $ms^{-1}$

**10** Two molecules of a gas have speeds of $9 \times 10^6$ m/s and $1 \times 10^6$ m/s, respectively. What is the root mean square speed of these molecules?

(a) $\sqrt{39} \times 10^6$ m/s (b) $6 \times 10^6$ m/s
(c) $\sqrt{41} \times 10^6$ m/s (d) $8 \times 10^6$ m/s

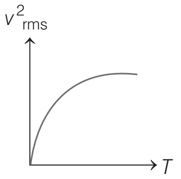
**11** For a gas at a temperature $T$ the root-mean-square velocity $v_{rms}$, the most probable speed $v_{mp}$ and the average speed $v_{av}$ obey the relationship

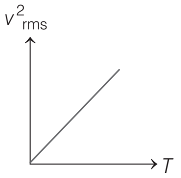
(a) $v_{av} > v_{rms} > v_{mp}$ (b) $v_{rms} > v_{av} > v_{mp}$
(c) $v_{mp} > v_{av} > v_{rms}$ (d) $v_{mp} > v_{rms} > v_{av}$

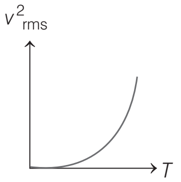
**12** Root mean square speed of the molecules of ideal gas is $v$. If pressure is increased two times at constant temperature, then the rms speed will become

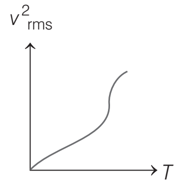
(a) $\frac{v}{2}$ (b) $v$ (c) $2v$ (d) $4v$

**13** The curve between absolute temperature and $v^2_{rms}$ is

(a)


(b)


(c)


(d)


**14** The degrees of freedom of a molecule of a triatomic gas are

(a) 2 (b) 4 (c) 6 (d) 8

**15** The temperature of argon, kept in a vessel, is raised by 1°C at a constant volume. The total heat supplied to the gas is a combination of translational and rotational energies. Their respective shares are

(a) 60% and 40% (b) 40% and 60%
(c) 50% and 50% (d) 100% and 0%

**16** One mole of ideal monoatomic gas ($\gamma$ =5/3) is mixed with one mole of diatomic gas ($\gamma$ =7/5). What is $\gamma$ for the mixture? ($\gamma$ denotes the ratio of specific heat at constant pressure, to that at constant volume)

(a) 3/2 (b) 23/15 (c) 35/23 (d) 4/3

**17** The ratio of specific heats ($\gamma$) of an ideal gas is given by

(a) $\frac{1}{1-\frac{R}{C_p}}$ (b) $1+\frac{R}{C_V}$ (c) $\frac{C_p}{C_p - R}$ (d) All of these

**18** The molar specific heat at constant pressure of an ideal gas is (7/2)$R$. The ratio of specific heat at constant pressure to that at constant volume is

(a) 7/5 (b) 8/7 (c) 5/7 (d) 9/7

**19** The heat required to increase the temperature of 4 moles of a monoatomic ideal gas from 273 K to 473 K at constant volume is

(a) 200 $R$ (b) 400 $R$
(c) 800 $R$ (d) 1200 $R$

**20** 4.0 g of a gas occupies 22.4 L at NTP. The specific heat capacity of the gas at constant volume is 5.0 $JK^{-1}$ $mol^{-1}$. If the speed of sound in this gas at NTP is 952 $ms^{-1}$, then the heat capacity at constant pressure is (Take gas constant $R = 8.3$ $JK^{-1}$ $mol^{-1}$)

➜ CBSE AIPMT 2015

(a) 8.0 $JK^{-1}$ $mol^{-1}$ (b) 7.5 $JK^{-1}$ $mol^{-1}$
(c) 7.0 $JK^{-1}$ $mol^{-1}$ (d) 8.5 $JK^{-1}$ $mol^{-1}$

**21** The ratio of the specific heats $\frac{C_p}{C_V} = \gamma$ in terms of degrees of freedom ($n$) is given by ➜ CBSE AIPMT 2015

(a) $\left(1+\frac{1}{n}\right)$ (b) $\left(1+\frac{n}{3}\right)$ (c) $\left(1+\frac{2}{n}\right)$ (d) $\left(1+\frac{n}{2}\right)$

**22** The amount of heat energy required to raise to temperature of 1 g of helium at NTP, from $T_1K$ to $T_2K$ is

➜ NEET 2013

(a) $\frac{3}{8}N_ak_B(T_2 - T_1)$ (b) $\frac{3}{2}N_ak_B(T_2 - T_1)$
(c) $\frac{3}{4}N_ak_B(T_2 - T_1)$ (d) $\frac{3}{4}N_ak_B\left(\frac{T_2}{T_1}\right)$

**23** When an ideal monoatomic gas is heated at constant pressure, fraction of heat energy supplied which increases the internal energy of gas, is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$
(c) $\frac{3}{7}$ (d) $\frac{3}{4}$

**24** For the specific heat of 1 mole of an ideal gas at constant pressure ($C_P$) and at constant volume ($C_v$), which is correct

(a) $C_P$ of hydrogen gas is $\frac{5}{2}R$ (b) $C_V$ of hydrogen gas is $\frac{7}{2}R$
(c) $H_2$ has very small values of $C_P$ and $C_V$
(d) $C_P - C_V = 1.99$ cal/mole-K for $H_2$

**25** If the internal energy of $n_1$ moles of He at temperature 10 T is equal to the internal energy of $n_2$ mole of hydrogen at temperature 6T. The mole of $\frac{n_1}{n_2}$ is

(a) $\frac{3}{5}$ (b) 2 (c) 1 (d) $\frac{5}{3}$

**26** 40 calories of heat is needed to raise the temperature of 1 mole of an ideal monoatomic gas from 20°C to 30°C at a constant pressure. The amount of heat required to raised its temperature over the same interval of a constant volume ($R = 2$ calorie $mole^{-1}K^{-2}$) is

(a) 20 cal (b) 40 cal (c) 60 cal (d) 80 cal

**27** A gas mixture consists of 2 moles of $O_2$ and 4 moles of Ar at temperature $T$. Neglecting all vibrational modes, the total internal energy of the system is ➜ NEET 2017

(a) 4$RT$ (b) 15$RT$ (c) 9$RT$ (d) 11$RT$

**28** One mole of an ideal monatomic gas undergoes a process described by the equation $pV^3 =$ constant. The heat capacity of the gas during this process is ➜ NEET 2016

(a) $\frac{3}{2}R$ (b) $\frac{5}{2}R$ (c) $2R$ (d) $R$

**29** The molar specific heats of an ideal gas at constant pressure and volume are denoted by $C_p$ and $C_V$ respectively. If $\gamma = \frac{C_p}{C_V}$ and $R$ is the universal gas constant, then $C_V$ is equal to ➜ NEET 2013

(a) $\frac{1+\gamma}{1-\gamma}$ (b) $\frac{R}{(\gamma - 1)}$ (c) $\frac{(\gamma - 1)}{R}$ (d) $\gamma R$

## DAY PRACTICE SESSION 2

# PROGRESSIVE QUESTIONS EXERCISE

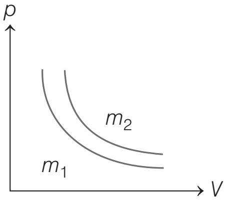
**1** The mean free path of molecules of a gas, (radius $r$) is inversely proportional to ➜ CBSE-AIPMT 2014

(a) $r^3$ (b) $r^2$ (c) $r$ (d) $\sqrt{r}$

**2** The equation of state for 5g of oxygen at a pressure $p$ and temperature $T$, when occupying a volume $V$, will be

(a) $pV = \left(\frac{5}{32}\right)RT$ (b) $pV = 5RT$
(c) $pV = \left(\frac{5}{2}\right)RT$ (d) $pV = \left(\frac{5}{16}\right)RT$

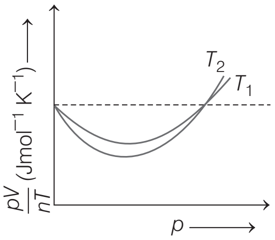
**3** Two different isotherms representing the relationship between pressure $p$ and volume $V$ at a given temperature of the same ideal gas are shown for masses $m_1$ and $m_2$, then



(a) $m_1 > m_2$
(b) $m_1 = m_2$
(c) $m_1 < m_2$
(d) Nothing can be predicted

**4** The figure below shows, the plot of $\frac{pV}{nT}$ *versus* $p$ for oxygen gas at two different temperatures.

Read the following statements concerning the above curves:



I The dotted line corresponds to the ideal gas behaviour

II $T_1 > T_2$

III The value of $\frac{pV}{nT}$ at the point, where the curves meet on the $Y$-axis is the same for all gases

Which of the above statements is true?

(a) Only (I) (b) Both (I) and (II)
(c) All of these (d) None of these

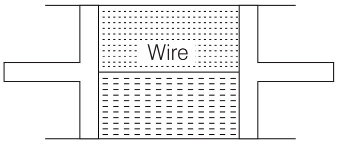
**5** The temperature at which the root mean square speed of a gas will be half its value at 0°C is (assume the pressure remains constant)

(a) – 86.4°C (b) – 204.75°C
(c) – 104.75°C (d) – 68.25°C

**6** Let $A$ and $B$ be two gases and given : $\frac{T_A}{M_A} = 4 \cdot \frac{T_B}{M_B}$, where, $T$ is the temperature and $M$ is molecular mass. If $C_A$ and $C_B$ are the rms speed, then the ratio $\frac{C_A}{C_B}$ will be equal to

(a) 2 (b) 4 (c) 1 (d) 0.5

**7** A cylindrical tube of uniform cross-sectional area $A$ is fitted with two air tight frictionless pistons. The piston are connected to each other by a metallic wire. Initially, the pressure of the gas is $p_0$ and temperature is $T_0$, atmospheric pressure is also $p_0$. Now, the temperature of the gas is increased to $2T_0$, the tension of the wire will be



(a) $2p_0A$ (b) $p_0A$ (c) $\frac{p_0A}{2}$ (d) $4p_0A$

**8** Two vessels separately contain two ideal gases $A$ and $B$ at the same temperature, the pressure of $A$ being twice that of $B$. Under such conditions, the density of $A$ is found to be 1.5 times the density of $B$. The ratio of molecular weight of $A$ and $B$ is ➜ **CBSE AIPMT 2015**

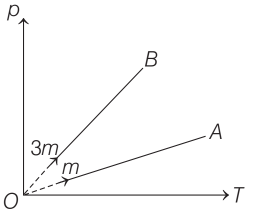
(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) 2 (d) $\frac{1}{2}$

**9** If temperature of the atmosphere varies with height as $T = (T_0 - ah)$, where $a$ and $T_0$ are positive constants, then the pressure as a function of height $h$ is (assume atmospheric pressure at sea level ($h = 0$) is $p_0$ and molecule mass $M$ of the air and acceleration due to gravity $g$ be constant)

(a) $p = p_0\left(\frac{T_0 - ah}{T_0}\right)^{Mg/Ra}$ (b) $p = p_0\left(\frac{T_0 - ah}{T_0}\right)^{2Mg/Ra}$

(c) $p = p_0\left(\frac{T_0 - ah}{T_0}\right)^{3Mg/Ra}$ (d) $p = p_0\left(\frac{T_0 - ah}{T_0}\right)^{4Mg/Ra}$

**10** Two different masses $m$ and $3m$ of an ideal gas are heated separately in a vessel of constant volume. The pressure $p$ and absolute temperature $T$ graphs for these two cases are shown in the figure as $A$ and $B$. The ratio of slopes of curves $B$ to $A$ is



(a) 3 : 1 (b) 1 : 3 (c) 9 : 1 (d) 1 : 9

**11** The quantity of heat required to raise one mole through one degree kelvin for a monoatomic gas at constant volume is

(a) $\frac{3}{2}R$ (b) $\frac{5}{2}R$ (c) $\frac{7}{2}R$ (d) $4R$

**12** Two mole of oxygen is mixed with eight mole of helium. The effective specific heat of the mixture at constant volume

(a) $1.3R$ (b) $1.4R$ (c) $1.7R$ (d) $1.9R$

**13** At 10°C the value of the density of a fixed mass of an ideal gas divided by its pressure is $x$. At 110°C this ratio is ➜ **CBSE AIPMT 2008**

(a) $x$ (b) $\frac{383}{283}x$ (c) $\frac{10}{110}x$ (d) $\frac{283}{383}x$

**14** A cylinder contains 20 kg of $N_2$ gas ($M$ = 28 kg kmol$^{-1}$) at a pressure of 5 atm. The mass of hydrogen ($M$ = 2 kg kmol$^{-1}$) at a pressure of 3 atm contained in the same cylinder at same temperature is

(a) 1.08 kg (b) 0.86 kg
(c) 0.68 kg (d) 1.68 kg

**15** At what temperature will the rms speed of oxygen molecules become just sufficient for escaping from the earth's atmosphere?
(Take, mass of oxygen molecule, $m = 2.76 \times 10^{-26}$ kg, Boltzmann's constant $k_B = 1.38 \times 10^{-23}$ J K$^{-1}$) ➜ **NEET 2018**

(a) $5.016 \times 10^4$ K (b) $8.326 \times 10^4$ K
(c) $2.508 \times 10^4$ K (d) $1.254 \times 10^4$ K

# ANSWERS

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| SESSION 1 | **1** (d) | **2** (c) | **3** (c) | **4** (b) | **5** (c) | **6** (c) | **7** (a) | **8** (c) | **9** (c) | **10** (c) |
| | **11** (b) | **12** (b) | **13** (b) | **14** (c) | **15** (d) | **16** (a) | **17** (c) | **18** (a) | **19** (d) | **20** (a) |
| | **21** (c) | **22** (a) | **23** (b) | **24** (d) | **25** (c) | **26** (a) | **27** (d) | **28** (d) | **29** (b) | |
| SESSION 2 | **1** (b) | **2** (a) | **3** (c) | **4** (c) | **5** (b) | **6** (a) | **7** (b) | **8** (b) | **9** (a) | **10** (a) |
| | **11** (a) | **12** (c) | **13** (d) | **14** (b) | **15** (b) | | | | | |

# Hints and Explanations

## SESSION 1

**1** If pressure of a given mass of the gas is kept constant, then

$$V \propto T$$

$$\Rightarrow \quad \frac{V}{T} = \text{constant} \Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\Rightarrow \quad V_2 = V_1\left(\frac{T_2}{T_1}\right)$$

Here, $T_1 = 273 + 27 = 300\,\text{K}$

$T_2 = 273 + 327 = 600\,\text{K}$

But $V_1 = 100\,\text{cc}$

$$V_2 = V_1\left(\frac{600}{300}\right)$$

$$V_2 = 2V_1$$

$$V_2 = 2 \times 100 = 200\,\text{cc}$$

**2**
$$pV = nRT$$

$$\Rightarrow \quad p \propto nT \quad [\because V \text{ and } R = \text{constants}]$$

$$\Rightarrow \quad \frac{p_2}{p_1} = \frac{n_2}{n_1} \times \frac{T_2}{T_1}$$

$$\Rightarrow \quad \frac{p_{\text{He}}}{p} = \frac{1}{1} \times \frac{2T}{T} \Rightarrow p_{\text{He}} = 2p$$

**3** $V$-$T$ graph is a straight line passing through origin.

Hence, process is isobaric.

$$V = \left(\frac{nR}{p}\right)T$$

$$\text{Slope} = \frac{nR}{p}$$

Slope of 2 > Slope of 1

$$\therefore \quad p_2 < p_1$$

**4**
$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$$\text{or} \quad V_2 = \frac{p_1V_1T_2}{p_2T_1} = \frac{1 \times 500 \times (273 - 3)}{0.5 \times (273 + 27)}$$

$$= \frac{1 \times 500 \times 270}{0.5 \times 300}$$

$$V_2 = 900\ \text{m}^3$$

**5** Mean kinetic energy of molecules depends upon temperature only. For $O_2$ it is same as that of $H_2$ at the same temperature of −73° C.

**6** The relationship between $p$, $V$ and $E$ for a gas is $pV = \frac{3}{2}E$

**7**
$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow \frac{v_s}{400} = \sqrt{\frac{273 + 227}{273 + 27}} = \sqrt{\frac{5}{3}}$$

$$\frac{v_s}{400} = 400\sqrt{\frac{5}{3}} = 516\ \text{m/s}$$

**8** Speed of sound in a gas $v = \sqrt{\frac{\gamma RT}{M}}$

$$\frac{v_{N_2}}{v_{\text{He}}} = \sqrt{\frac{\gamma_{N_2}}{\gamma_{\text{He}}} \times \frac{M_{\text{He}}}{M_{N_2}}}$$

$$= \sqrt{\frac{(7/5)}{(5/3)} \times \frac{4}{28}} = \frac{\sqrt{3}}{5}$$

**9** $v_{\text{rms}} \propto \sqrt{T}$. If temperature is doubled rms speed will become $\sqrt{2}\ v_{\text{rms}}$ times.

$$\Rightarrow \quad v_{\text{rms}} = 200\sqrt{2}\ \text{ms}^{-1}$$

**10** By definition,

$v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2}{2}}$, where $v_1$ and $v_2$ are the individual velocities of the two particles.

Given, $v_1 = 9 \times 10^6\ \text{m/s}$

and $v_2 = 1 \times 10^6\ \text{m/s}$

$$\therefore \quad v_{\text{rms}} = \sqrt{\frac{(9 \times 10^6)^2 + (1 \times 10^6)^2}{2}}$$

$$= \sqrt{\frac{81 \times 10^{12} + 1 \times 10^{12}}{2}}$$

$$= \sqrt{\frac{(81 + 1) \times 10^{12}}{2}}$$

$$= \sqrt{\frac{82 \times 10^{12}}{2}}$$

$$= \sqrt{41} \times 10^6\ \text{m/s}$$

**11** As we know that

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}},\ v_{\text{mp}} = \sqrt{\frac{2KT}{m}},$$

$$v_{\text{av}} = \sqrt{\frac{8kT}{\pi m}}$$

Hence, $v_{\text{rms}} > v_{\text{av}} > v_{\text{mp}}$

**12** Root mean square speed,

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3p}{\rho}}$$

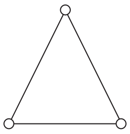
On increasing the pressure by the two times, the density also increases in the same ratio, so that rms speed remains same.

**13** As we know that,

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}},\ v_{\text{rms}}^2 \propto T$$

Hence, option (b) is correct.

**14** **Non-linear triatomic molecule** In this molecule, the three atoms are present at the vertices of triangle.



Here, $A = 3$ and $R = 3$

$$\therefore \quad N = 3 \times 3 - 3 = 6$$

So, a non-linear triatomic molecule has six degree of freedom.

**15** Argon is a monoatomic gas, so it has only translational energy.

**16**
$$\gamma_{\text{mix}} = \frac{\dfrac{\mu_1\gamma_1}{\gamma_1 - 1} + \dfrac{\mu_2\gamma_2}{\gamma_2 - 1}}{\dfrac{\mu_1}{\gamma_1 - 1} + \dfrac{\mu_2}{\gamma_2 - 1}}$$

$$= \frac{\dfrac{1 \times \frac{5}{3}}{\left(\frac{5}{3} - 1\right)} + \dfrac{1 \times \frac{7}{5}}{\left(\frac{7}{5} - 1\right)}}{\dfrac{1}{\left(\frac{5}{3} - 1\right)} + \dfrac{1}{\left(\frac{7}{5} - 1\right)}} = \frac{3}{2}$$

**17** $C_p - C_V = R$ ...(i)

On dividing Eq. (i) by $C_p$, we get

$$1-\frac{C_V}{C_p}=\frac{R}{C_p} \text{ or } 1-\frac{1}{\gamma}=\frac{R}{C_p}\left[\because \frac{C_p}{C_V}=\gamma\right]$$

$$\Rightarrow \frac{1}{\gamma}=1-\frac{R}{C_p} \text{ or } \gamma=\frac{C_p}{C_p-R}$$

**18** We have given molar specific heat at constant pressure,

$$C_p=\frac{7}{2}R$$

Mayer's relation is

$$C_V=C_p-R=\frac{5}{2}R$$

Hence, required ratio is

$$\gamma=\frac{C_p}{C_V}=\frac{\left(\frac{7}{2}\right)R}{\left(\frac{5}{2}\right)R}=\frac{7}{5}$$

**19** Specific heat for a monoatomic gas,

$$C_V=\frac{3}{2}R$$

$$\therefore \text{ Heat, } dQ=\mu C_V\Delta T$$

$$=4\times\frac{3}{2}\times R\,(473-273)`$$

$[\because \mu = 4]$

$$=4\times\frac{3}{2}\times R\times 200$$

$$\therefore \quad dQ=4\times 300R$$

$$=1200\,R$$

**20** Given, $M = 4$ gm, $V = 22.4$ L,

$$C_V = 5\ \text{JK}^{-1}\text{mol}^{-1}$$

$$v_{\text{sound}} = 952\ \text{m/s},\ C_p = ?$$

As, velocity of sound,

$$v_{\text{sound}}=\sqrt{\frac{\gamma pV}{M}} \Rightarrow \gamma=\frac{M}{pV}v^2_{\text{sound}}=\frac{C_p}{C_V}$$

So, heat capacity at constant pressure,

$$C_p=C_V\left[\frac{M}{pV}\right]v^2_{\text{sound}}$$

$$=5\left[\frac{4\times10^{-3}}{10^5\times22.4\times10^{-3}}\right](952)^2$$

$$=\frac{20}{22.4}\times(952)^2\times10^{-5}$$

$$=809200\times10^{-5}=8.09\ \text{J/mol K}$$

$$\simeq 8.0\ \text{JK}^{-1}\text{mol}^{-1}$$

**21** $C_V=\frac{n}{2}R$

$$\because \quad C_p=C_V+R=\frac{n}{2}R+R$$

$$\therefore \quad C_p=\left(\frac{n}{2}+1\right)R$$

$$\text{Now, } \gamma=\frac{C_p}{C_V}=\frac{\left(\frac{n}{2}+1\right)R}{\frac{n}{2}R}=\frac{n+2}{n}$$

$$\gamma=1+\frac{2}{n}$$

**22** We know that, $Q=\frac{f}{2}nR\Delta T$

$\therefore$ Amount of heat required,

$$Q=\frac{3}{2}\times\frac{1}{4}\times k_B N_a \Delta T$$

$$=\frac{3}{8}N_a k_B(T_2-T_1)$$

**23** For monoatomic gas, $\gamma=\frac{C_p}{C_V}=\frac{5}{3}$

We know that,

$$\Delta Q=\mu C_p\Delta T \text{ and } \Delta U=\mu C_V \Delta T$$

$$\frac{\Delta U}{\Delta Q}=\frac{C_V}{C_p}=\frac{3}{5}$$

i.e. fraction of heat energy to increase the internal energy be $\frac{3}{5}$.

**24** For any gas

$$C_P-C_V=1.99\approx 2\ \text{cal mol}^{-1}\text{K}^{-1}$$

**25** $n_1 C_{V_1}\,\Delta T_1 = n_2 C_{V_2}\,\Delta T_2$

$$n_1\times\frac{3}{2}R\times10=n_2\times\frac{5}{2}R\times6$$

$$\frac{n_1}{n_2}=1$$

**26** At constant pressure,

$$(\Delta Q)_p=\mu C_p\,\Delta T$$

$$=1\times C_p\times(30-20)=40$$

$$C_p=4\ \text{cal mol}^{-1}\text{K}^{-1}$$

$$C_V=C_p-R$$

$$=4-2=2\ \text{cal mol}^{-1}\text{K}^{-1}$$

Now, $(\Delta Q)_V=\mu c_V\Delta T=1\times2\times(30-20)$

$$=20\ \text{cal}$$

**27** Total internal energy of system

= Internal energy of oxygen molecules + Internal energy of argon molecules

$$=\frac{f_1}{2}n_1RT+\frac{f_2}{2}n_2RT$$

$$=\frac{5}{2}\times2RT+\frac{3}{2}\times4RT=11RT$$

**28** As we know that for polytropic process of index $\alpha$ specific heat capacity

$$=C_V+\frac{R}{1-\alpha}$$

$\because$ Process, $pV^3$ = constant $\Rightarrow \alpha = 3$

$$\therefore \quad C=C_V+\frac{R}{1-\alpha}$$

$$\text{where, } C_V=\frac{fR}{2}$$

$$=\frac{fR}{2}+\frac{R}{1-3}$$

For monatomic gas, $f=3=\frac{3R}{2}$

$$\Rightarrow \quad C=\frac{3R}{2}-\frac{R}{2}=R$$

**29** As we know that

$$C_p-C_V=R$$

$$C_p=R+C_V$$

$$\text{and} \quad \frac{C_p}{C_V}=\gamma \quad \text{(given)}$$

$$\text{So,} \quad \frac{R+C_V}{C_V}=\gamma$$

$$\Rightarrow \gamma C_V=R+C_V\Rightarrow\gamma C_V-C_V=R$$

$$\Rightarrow \quad C_V=\frac{R}{\gamma-1}$$

## SESSION 2

**1** $\lambda=\frac{1}{\lambda d^2 n\sqrt{2}}=\frac{1}{4\pi r^2 n\sqrt{2}}\Rightarrow\lambda\propto\frac{1}{r^2}$

**2** Number of moles,

$$n=\frac{m}{\text{molecular weight}}=\frac{5}{32}$$

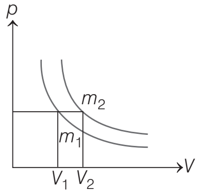
So, from ideal gas equation,

$$pV=nRT \Rightarrow pV=\frac{5}{32}RT$$

**3** $pV=nRT=\frac{m}{M}RT$

For first graph,

$$p=\frac{m_1}{M}\frac{RT}{V_1} \quad \text{...(i)}$$



For IInd graph,

$$p=\frac{m_2}{M}\frac{RT}{V_2} \quad \text{...(ii)}$$

On equating Eqs. (i) and (ii), we get

$$\frac{m_1}{m_2}=\frac{V_1}{V_2}\Rightarrow m\propto V$$

$$\text{As,} \quad V_2>V_1\Rightarrow m_1<m_2$$

**4** (i) The dotted line in the diagram shows that there is no deviation in the value of $\frac{pV}{nT}$ for different temperatures $T_1$ and $T_2$ for increasing pressure so, this gas behaves ideally.

(ii) At high temperature, the deviation of the gas is less and at low temperature the deviation of gas is more. In the graph, deviation for $T_2$ is greater than for $T_1$.

(iii) The two curves intersect at dotted line, so the value of $\frac{pV}{nT}$ at that point on the $y$-axis is same for all gases.

**5** $v_{\text{rms}} \propto \sqrt{T}$, rms speed will remain half, if temperature becomes $\frac{1}{4}$th or $\left(\frac{273}{4} - 273\right)^\circ$C or – 204.75°C.

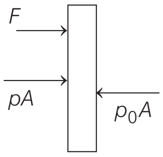
**6** $\frac{T_A}{M_A} = 4\,\frac{T_B}{M_B} \Rightarrow \sqrt{\frac{T_A}{M_A}} = 2\sqrt{\frac{T_B}{M_B}}$

$$\Rightarrow \quad \sqrt{\frac{3RT_A}{M_A}} = 2\sqrt{\frac{3RT_B}{M_B}}$$

$$\Rightarrow \quad C_A = 2C_B$$

$$\Rightarrow \quad \frac{C_A}{C_B} = 2$$

**7** Volume of the gas is constant
i.e. $V$ = constant



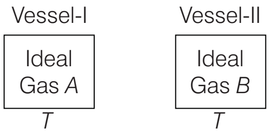
$\therefore\ p \propto T$
i.e. pressure will be
doubled temperature is doubled

$$\therefore \quad p = 2p_0$$

Let $F$ be the tension in the wire. Then, equilibrium of any one piston gives

$$F = (p - p_0)A$$
$$= (2p_0 - p_0)A = p_0A$$

**8**



$\rho_A = 1.5\,\rho_B$
$p_A = 2p_B$

According to ideal gas equation, we have

Pressure, $p = \frac{\rho RT}{M}$, where $M$ is molecular weight of ideal gas.

Such that, $\frac{p}{\rho} = \frac{RT}{M} \quad \Rightarrow M = \frac{\rho RT}{P}$

where, $R$ and $T$ are constants.

So, $M \propto \frac{\rho}{p}$

$$\Rightarrow \frac{M_A}{M_B} = \frac{\rho_A}{\rho_B} \times \frac{p_B}{p_A} = 1.5 \times \frac{1}{2}$$
$$= 0.75 = \frac{3}{4}$$

**9** $\frac{dp}{dh} = -\rho g = -\left(\frac{pM}{RT}\right)g$

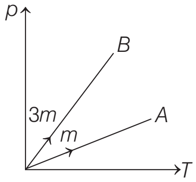
$$= -\left[\frac{pM}{R\,(T_0 - ah)}\right]g$$

$$\frac{dp}{p} = -\left(\frac{Mg}{R}\right)\frac{dh}{T_0 - ah}$$

$$\int_{p_0}^{p} \frac{dp}{p} = -\left(\frac{Mg}{R}\right)\int_0^h \frac{dh}{(T_0 - ah)}$$

$$p = p_0\left(\frac{T_0 - ah}{T_0}\right)^{\frac{Mg}{Ra}}.$$

**10** For a gas, $pV = nRT = \frac{m}{M}RT$



Slope of graph $A$,

$$\left(\frac{p}{T}\right) = \frac{m}{M}\,\frac{R}{V} \qquad \text{...(i)}$$

Similarly, slope of graph $B$,

$$\frac{p}{T} = \frac{3m}{M}\,\frac{R}{V} \qquad \text{...(ii)}$$

$$\therefore \quad \frac{\text{Slope of curve } B}{\text{Slope of curve } A} = \frac{\frac{3m}{M}\,\frac{R}{V}}{\frac{m}{M}\,\frac{R}{V}} = \frac{3}{1}$$

**11** $(\Delta Q)_V = \mu C_V \Delta T$

$(\Delta Q)_V = 1 \times C_V \times 1$
$= C_V$

For monoatomic gas,

$$C_V = \frac{3}{2}R$$

$$\therefore \quad (\Delta Q)_V = \frac{3}{2}R$$

**12** For mixture of gases, let specific heat be $C_V$.

$$C_V = \frac{n_1(C_V)_1 + n_2(C_V)_2}{n_1 + n_2}$$

$$= \frac{2 \times \frac{5R}{2} + 8 \times \frac{3R}{2}}{2 + 8} = \frac{17R}{10} = 1.7R$$

**13** Ideal gas equation

$$pV = nRT$$

$$\frac{pV}{m} = \frac{1}{M}RT \qquad \left(\because n = \frac{m}{M}\right)$$

$$\frac{p}{\rho} = \frac{RT}{M} \qquad \left(\because \frac{V}{m} = \frac{1}{\rho}\right)$$

$$\therefore \quad \frac{\rho}{p} \propto \frac{1}{T}$$

Molecular mass $M$ and universal gas constant $R$ remains same for a gas.
So, for two different situations i.e. at two different temperatures and densities

$$\therefore \quad \frac{\rho_1/p_1}{\rho_2/p_2} = \frac{T_2}{T_1} \Rightarrow \frac{x}{(\rho_2/p_2)} = \frac{383\ \text{K}}{283\ \text{K}}$$

$$\Rightarrow \quad \frac{\rho_2}{p_2} = \frac{283}{383}\,x$$

**14** $\frac{V}{T} = \frac{nR}{p}$

$V$ and $T$ for both cases is same.

Hence, $\frac{n_1}{p_1} = \frac{n_2}{p_2}$ or $\frac{m_1}{p_1M_1} = \frac{m_2}{p_2M_2}$

or $\quad m_2 = \frac{p_2M_2}{p_1M_1}\,m_1$

$$= \frac{(3)(2)}{(5)(28)}\,20 = 0.86\ \text{kg}$$

**15** (b) The minimum velocity with which the body must be projected vertically upwards, so that it could escape from the Earth's atmosphere, is its escape velocity ($v_e$).

As, $v_e = \sqrt{2gR}$

Substituting the value of $g$ $(9.8\ \text{ms}^{-2})$ and radius of Earth ($R = 6.4 \times 10^6$ m), we get

$$v_e = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$
$$\cong 11.2\ \text{km s}^{-1}$$
$$= 11200\ \text{m s}^{-1}$$

Let the temperature of molecule be $T$ when it attains $v_e$.

According to the question,

$$v_{\text{rms}} = v_e$$

where, $v_{\text{rms}}$ is the rms speed of the oxygen molecule.

$$\Rightarrow \quad \sqrt{\frac{3k_BT}{m_{O_2}}} = 11.2 \times 10^3$$

$$\text{or} \quad T = \frac{(11.2 \times 10^3)^2\,(m_{O_2})}{(3k_B)}$$

Substituting the given values, i.e.,

$$R_B = 1.38 \times 10^{-23}\ \text{JK}^{-1} \text{ and}$$

$$m_{O_2} = m = 2.76 \times 10^{-26}\ \text{kg}$$

We get,

$$T = \frac{(11.2 \times 10^3)^2\,(2.76 \times 10^{-26})}{(3 \times 1.38 \times 10^{-23})}$$
$$= 8.3626 \times 10^4\ \text{K}$$